Published by the
Office of Real Estate Research
University of Illinois at Urbana-Champaign
Winter 1998

Illinois Real Estate Letter

Weird Finance

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The beginning finance student learns about *duration* in the first course that provides any in-depth coverage of bonds and other fixed-income securities. The class is shown a formula, taught how to use it in calculating duration, and told that duration provides a quick, simple way to measure the interest rate-related risks faced by an investor who holds a traditional corporate or government bond. However, most students never learn of the peculiar – perhaps we should characterize them as *weird* – things that happen when the duration idea is applied to more exotic types of fixed-income securities.

In this article, we explore a few of these more exotic securities and discuss some of the weird things that happen when we try to analyze such financial instruments with the duration tool. We begin the next section with a more formal definition and review of duration, and how it is calculated. Then we proceed to discuss a particular group of exotic instruments known as interest-sensitive fixed-income securities. The discussion concludes with some suggestions on how investors can use duration in managing risks that relate to changing interest rates.

The Nature of Fixed-Income Securities

In general, a fixed-income security, such as a bond or mortgage note, is an instrument that provides its holder with a return *of* the principal advanced plus a pre-determined rate of return *on* that principal, but will not pay back any more or less. In the simplest cases, the rate of return is unchanging in nominal (unadjusted for inflation) terms, and is received for a definite stated time period.

In a slightly more complex case, the rate of return might vary in a pre-determined manner with market conditions, perhaps to keep pace with inflation. Alternatively, the time period over which returns are received could be shorter than the instrument's stated life; the user of the money can pay the principal back early, often by borrowing from another investor at a lower interest rate, if the instrument is *callable*. (Securities that are *not* of the fixed-income variety include equity claims on corporate profits or real estate, the holders of which receive *residual* sums that are uncertain both in amount and in length of the payment period, because they directly reflect the performance of the underlying assets.)

The fixed-income security holder receives a return only on the amount of principal that has not yet been repaid. Principal is recouped slowly, either systematically over the contract life, as with a mortgage note; or in a lump sum at the end of the contract period, as with a traditional corporate or government bond (although the process speeds up when a callable security is *called*). If this slow repayment becomes inconsistent with the investor's objectives, the fixed-income security can be sold to another investor in

Tenth Anniversary Issue

Actually, the premier *ORER Letter* was distributed in the Fall of 1987. That first issue was undertaken by Peter Colwell on a wing and a prayer, and a primitive desktop publishing package. But the great success of that initial attempt provided proof of the viability of a publication devoted to bridging the gap between the academy and industry. The Winter 1988 issue was the first one we produced knowing that we were "in it for the long haul." There have been ups and downs, of course; our quarterly printing schedule, to which we adhered faithfully from 1988 through 1991, had to be curtailed in 1992 with the state's taking of money from the Real Estate Research and Education Fund and the Real Estate Recovery Fund, which help to finance ORER's activities. But after keeping our foundering craft afloat with semiannual issues from 1992 to 1996 (and adopting the more descriptive *Illinois Real Estate Letter* moniker in 1993), we returned to quarterly offerings with the Winter 1997 edition. We hope that our readers enjoy this *32nd* issue of our publication, and that we will be bringing you provocative commentary and analysis on real estate issues affecting Illinois citizens for many years to come. Thank you for your continued interest and support.

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Winter 1998 – Volume 12, Number 1

Illinois Real Estate Letter is published by the Office of Real Estate Research at the University of Illinois at Urbana-Champaign
http://www.cba.uiuc.edu/orer/orer.htm

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the *secondary market*. The price received depends on the supply of and demand for the security, and on such *fundamental* factors as the principal balance remaining and prevailing market interest rates.

A fixed-income security's secondary market value changes if the rate of return that investors require for accepting the attendant risks becomes different from the return represented by the cash flows. For example, if a $1,000 bond has a *coupon* rate of 10%, its holder receives $100 per year (typically two $50 semiannual payments) in interest. But if economic conditions or the financial strength of the issuer change such that investors require a return other than 10%, the bond's value (as evidenced by its market price) would have to change so the $50 interest payments and $1,000 principal receipt would represent an appropriate dollar return.

We can compute the expected price P of a corporate bond that provides semiannual coupon interest payments of c_t, and has a face value (the loan principal, returned to the investor along with the final coupon payment) of F, by *discounting* the coupon and principal payments to a *present value* at the market rate of return, or *yield*, r that investors expect based on their current assessment of risk:

$$P = \frac{c_1}{(1+r)^1} + \frac{c_2}{(1+r)^2} + \cdots + \frac{c_T + F}{(1+r)^T}.$$

From this equation, those who have mastered basic *time value of money* concepts will see that as the required market yield r rises, the present value of the c and F cash flows declines. Conversely, as the required interest rate falls, the present value of the bond's cash flows increases.

Because the bond's value rises when market interest rates fall and falls when market yields rise, we can conclude that a fundamental relationship involving simple fixed-income instruments is that value changes when interest rates change, but in the opposite direction. The rate of return that the market deems appropriate for a particular fixed-income investment reflects both the security issuer's ability to make promised payments, and general economic factors (including inflation expectations) that affect all market participants. In this discussion, our focus is on market-wide yield changes that affect the values of all fixed-income investments.

What Is Duration?

Investors are highly concerned with the risks that possible interest rate changes impose on holders of fixed-income securities. Because bond prices rise or fall as market yields change, investors need a measure of how much a security's price can be expected to change in response to a given rise or fall in those market rates. The mechanics of discounting, based on the discount factor $1/(1+r)^n$, causes bonds with longer maturities to experience greater changes in value than those with shorter maturities when interest rates change by a particular amount; for cash flows distant in time, the large magnitude of exponent n has a severe value impact. Since the market is populated with bonds having different maturities and coupons, investors need a measure that allows for direct comparisons across instruments.

In 1938, Frederick Macaulay introduced *duration* to help investors measure the price risk associated with changes in market interest rates. Macaulay's duration is a measure of the "term" of the bond; as we will see, it is also a measure of interest rate risk. Its magnitude is a weighted average of the time it takes for the various parts of a bond's cash flows – interest and the return of principal, or face value – to be received (more technically it is the bond's "weighted average term to maturity").

In the computation of Macaulay duration, the time when a cash flow is to be received is weighted by the present value of that cash flow. Thus duration (D) is computed in this manner as

$$D = \frac{1}{P}\left[\frac{1c_1}{(1+r)^1} + \frac{2c_2}{(1+r)^2} + \cdots + \frac{Tc_T}{(1+r)^T}\right],$$

an equation in which T is the number of periods when cash flows are received. Using this formula, we find duration to be a positive number expressed in years (or whatever time periods are signified by the digits 1 through T in the respective numerators in the above formula).

In simplest terms, Macaulay duration provides a measure that combines the bond's time to maturity with interest rates. Factors affecting duration include the bond's maturity, its required yield, and its coupon interest rate (the "stated" rate, which equals the periodic interest payment as a proportion of the bond's face value). As the name suggests, this "duration" is related to maturity; it is a weighted average measure of the time it takes to receive *all* cash flows, whereas maturity is simply how long we have to wait until the *last* cash flow is received.

Macaulay duration is usually higher for a bond with a longer life, although it is always less than or equal to the bond's life, or maturity. (In the extreme, a *zero coupon bond* has duration = maturity, because all cash flows are received on the maturity date.) A bond with a higher, or a more frequent (*e.g.*, monthly), coupon payment has a lower Macaulay duration because relatively more of the cash flow total is received sooner. A higher yield also results in lower Macaulay duration, as the action of exponent *n* further reduces the weight on a distant time period. Higher measured duration usually is equated with greater risk.

Because Macaulay duration provides a measure of the weighted average length of the bond's payment stream, it is a more effective tool for ranking securities with different coupons than is simple maturity. Yet the duration idea is also useful to investors in determining the *price risk* of a bond's cash flows, since it can be used as a tool to approximate the price volatility of fixed income investments (the degree to which a bond's price is expected to change in response to a specified change in market yields). Thus, duration is used by risk managers to measure a particular bond's interest rate risk and as a tool to rank fixed-income securities generally according to interest rate risk. In other words, aside from Macaulay's time-based measure, duration can indicate the percentage change in price for a corresponding change in market interest rates.

A Simple Macaulay Duration Example
Consider a 10% corporate bond maturing in two years (with four semiannual coupon payments, as typically characterize corporate bonds in the US). The investor will receive the $1,000 face value at the end of the second year, along with the final interest payment. If the rate of return (yield) currently required by the market is 8% (4% semiannually),[1] the price and the Macaulay duration of this bond are computed, respectively, as

$$PV = \frac{50}{(1.04)^1} + \frac{50}{(1.04)^2} + \frac{50}{(1.04)^3} + \frac{1050}{(1.04)^4} = \$1{,}036.30$$

$$D = \frac{\dfrac{(1)50}{(1.04)^1} + \dfrac{(2)50}{(1.04)^2} + \dfrac{(3)50}{(1.04)^3} + \dfrac{(4)1050}{(1.04)^4}}{\$1{,}036.30} = 1.86\text{yrs}$$

(actually the solution shows 3.72 *periods*, but these semiannual periods convert to 1.86 years). To see the effect of a change in market interest rates on the price of this bond, suppose that the required yield rises to 9% annually. The expected percentage price change for a given change in market interest rates is computed as

$$\frac{dP}{P} = -\frac{D}{(1+r)}\, d(1+r)\ ,$$

an equation in which *d* is the symbol for change and $-D/(1+r)$ is referred to as *modified duration*. Substituting, we find

$$\frac{dP}{P} = -\frac{1.86}{1.09}\,(.01) = .017\,.$$

Thus we see that a 1% increase in market interest rates (from 8% to 9%) results in a 1.7% decline in the bond's value.

Now we can examine the effects of changes in a bond's yield and maturity on its Macaulay duration. Consider a 10% coupon bond with 30 years to maturity. If the required yield is 8%, its price is $1,226.23 and its Macaulay duration is 11.411 years; at a 12% yield its price is $838.39 and Macaulay duration is 8.695 years. (A 10% coupon bond is worth its $1,000 face value if the required yield is 10%.) Table 1 provides a comparison of duration, as computed by Macaulay's formula, for a simple 10% coupon bond selling at a premium (8% yield) or discount (12% yield) for three selected maturities. Table 1 shows this duration measure to increase with the bond's maturity, and to fall as required market yields rise.

To summarize, duration is a measure with several important uses. Macaulay's approach to duration provides a time dimension that allows bond investors to compare the risks associated with bonds that have differing characteristics. A bond with lower Macaulay duration puts money back in the lender's hands more quickly, on average. But the duration concept also allows an investor to estimate the price change that should be expected when market yields change by a specified amount. Finally, as we discuss later, duration also allows an investor to create *hedging* strategies to minimize the risks that changes in market interest rates impose on fixed-income portfolios.

Table 1

Price and Duration for a 10%, Non-Callable Coupon Bond

		Years		
Yield: 8%		2	5	30
	Price	$1,036.30	$1,081.11	$1,226.23
	Duration	1.864	4.095	11.411
Yield: 12%		2	5	30
	Price	$965.35	$926.40	$838.39
	Duration	1.859	4.011	8.695

Duration on Complex Securities
The prior section presented the Macaulay formula for calculating duration for fixed-income securities with fixed, certain cash flow streams. It is unfortunate that this formula is *not* useful for dealing with securities whose cash flows are interest rate-sensitive. An interest rate-sensitive security is one for which the number or amount of the nominal cash flows depends on changes in required market yields. For example, a mortgage loan, or a mortgage-backed security, is rate-sensitive because the callability (the borrower's right to prepay the note) causes the investor's cash flows to depend on the future level of market interest rates. When interest rates fall, borrowers tend to repay their mortgage loans by refinancing to take advantage of the lower cost. Thus an investor expecting a stream of cash flows for a fixed period is faced with the *reinvestment risk* associated with the possible early return of principal. Rate-sensitive instruments create serious problems for investors, because both

duration and the cash flows themselves change when interest rates change.

Consider the pricing problem associated with a mortgage-backed security (MBS). If market interest rates decline, borrowers begin repaying their mortgage loans to take advantage of refinancing opportunities. Because the increased prepayments that accompany lower rates result in a decline in the MBS's duration, the MBS interest rate/price relationship does not always follow that observed for traditional, non-callable fixed-income securities (for which lower market rates cause higher durations, as shown in Table 1). This case of higher yields resulting in higher prices over a given range appears graphically as a pattern referred to as *negative convexity.* Figure 1 shows the relationship between price and required yield for a traditional non-callable bond, and for an MBS over the range of yields for which it displays negative convexity.

Securities characterized by negative convexity can sometimes pose puzzling duration questions. Think of what happens to a mortgage-backed security when market interest rates are far below the contract (coupon) rate on the notes in the MBS pool. In this situation, investors expect most loans in the pool to be repaid very quickly, causing duration to be very small. In fact, special types of MBSs with cash flows highly dependent on prepayment activity may actually have *negative durations* (a weird situation, in that Macaulay duration for a simple fixed-income security is always a positive magnitude). Examples of securities with negative durations include *interest-only strips* (IOs), mortgage *service flow* contracts, and *residual CMO tranches.*[2]

Figure 1

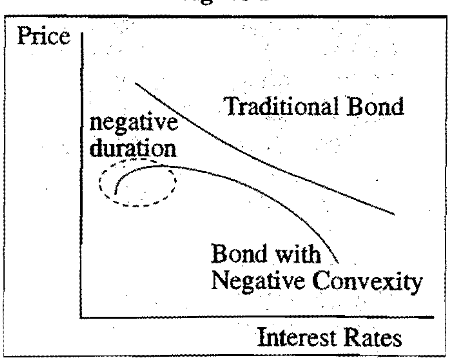

Table 2

Price, Duration, and Convexity of GNMAs
(November 8, 1985)

Coupon	Market Price	Model Price	Duration (Years)
7.5%	$84.38	$84.29	6.1
8.0%	$86.69	$87.01	6.0
9.0%	$91.09	$92.73	5.5
10.0%	$96.06	$97.38	5.2

Source: Jacob, David P., Graham Lord, and James A. Tilley, "Price, Duration and Convexity of Mortgage-Backed Securities," in *Mortgage-Backed Securities*, Frank J. Fabozzi (ed.), Chicago: Probus Publishing, 1987, p. 96 (Exhibit 11).

Weird Duration Example: IO Strips

To demonstrate the weird concept of negative duration, we consider the pricing of an interest-only strip. An IO is a security that consists of coupon payments only, with no principal received. In its most simple form, an IO is created when a mortgage loan payment is separated into its principal and interest components. The IO strip holder receives only the interest part of the loan payment, while the principal part of the payment is placed into a *principal-only* (PO) strip. We compute the price of the IO as

$$P_{IO} = \frac{c_1}{(1+r)^1} + \frac{c_2}{(1+r)^2} + \cdots + \frac{c_T}{(1+r)^T} .$$

In this equation the c_i are, as before, the coupon interest payments. For the IO strip, however, the number (and thus the total value) of coupon payments is unknown. If market interest rates fall, there is an increased probability that the borrower will prepay; as a result, the IO investor's coupon stream may be much shorter than had initially been expected. Thus the present value of the IO cash flows (the price that a buyer would pay in a secondary market transaction) will decline. However, if interest rates rise then the likelihood of prepayment falls, and the investor receives a longer stream of interest payments than had been expected. Thus, the IO's price will *rise* over some range of *increase* in market rates. And because the number of IO cash flows changes as interest rates change, IO prices are *highly* sensitive to changes in interest rates. Yet because IO strip price reaction to movements in market interest rates is opposite that of traditional fixed-income securities, IOs are extremely useful tools for portfolio hedging.[3]

Generalizing the duration definition to the case of the IO (or other interest rate-sensitive security) requires computing the change in the present value of the IO cash flows for various changes in market yields. Unfortunately, an equation for calculating duration in a Macaulay-like manner does not exist for interest rate-sensitive securities. Thus, several methods have been proposed for *approximating* durations for securities with uncertain cash flows. For example, Davidson (1987) has proposed five alternative methods for calculating the duration of a mortgage-backed security.[4] Jacob, Lord, and Tilley (1987) offer an example of calculating duration for MBSs using Davidson's "options approach."[5]

Table 2 provides selected results relating to Jacob, Lord, and Tilley's calculations of price and duration for a series of GNMA securities on a selected day in 1985. After calculating the price of each underlying mortgage loan twice (for two different interest rates), they define duration as *(-d(Price)/(d(i))*Price)*, or the negative of the change in price divided by the product of the price and the change in the interest rate. Using this definition to calculate duration, we can easily get a negative result. For example, because IO strip prices increase as interest rates rise, it follows that both the change in price and the change in the interest rate are positive, and thus duration is negative.

In order to make sense of negative duration, we must bear in mind that duration, as discussed earlier, has two meanings. First, it measures the weighted average of the timing of the cash flows. Second, it is an estimate of the bond's price volatility relative to changes in market interest rates. Under Macaulay's definition, there is no meaningful way to interpret a negative duration value. For example, it is nonsensical to state that the cash flows for a security with a duration of −1.2 years are received, on average, 1.2 years before the security's issue date. Rather, negative duration has meaning only when we use it in measuring the sensitivity of price changes to interest rate changes. Putting a negative duration value into the equation that embodies this second definition implies only that the rate-sensitive instrument's price will increase with an interest rate increase and decline when market rates drop. This counterintuitive result for IOs follows from the fact that as market interest rates rise, mortgage loan prepayments decline. Thus a rise in market rates will cause the investor to receive cash flows on an IO strip for a longer period than had been anticipated when the security was purchased, resulting in an increased price.

Or think of the situation in a different way. When an IO is first issued, an investor buys it for a price equal to the present value of cash flows she *expects* to receive. She knows that some prepayments will occur as borrowers sell their homes or refinance to withdraw equity, but she has no sure way to know how prepayments will be affected by refinancings that relate to changes in required market yields. In examining the range of possibilities, she factors in the chance that rates may soon drop, triggering a high level of refinancing and prepayment.

But if, shortly after her IO purchase, market rates *rise*, it becomes clear that she has overestimated prepayments and underestimated cash flows. If she then sells the IO in a secondary market transaction, the buyer will observe that high market interest rates (relative to the pooled loans' contract rate) render refinancing and prepayment unlikely, and thus will expect higher cash flows – and will be willing to pay a higher price for the IO strip – than had the seller.

It should be noted that negative durations will be encountered only within certain interest rate ranges. For example, once market yields rise above a given level the borrowers in the loan pool all will have decided not to refinance. Any further increase in rates will have no impact on prepayments, and thus will not cause the IO strip's price to rise further. In fact, it will cause the price to *fall*, as in the case of a more typical bond, because of the higher rate at which the expected cash flows will be discounted.

Mortgage Servicing Flow Contracts

Another interesting example of a fixed-income security with weird duration is the mortgage servicing flow contract. A mortgage-backed security is created when a financial organization combines many individual loans into a pool, and then sells claims (securities) on the pool's cash flows to investors. For example, an investment banking firm could collect 1,000 FHA loans issued at about the same time with contract rates of 10%, and issue 9.5% GNMA mortgage *pass-throughs* backed by this pool. The issuer collects payments and passes them on to the investors, keeping the difference between the 10% contract rate and the 9.5% pass-through rate as a fee.[6] This fee is known as the mortgage loan servicing flow contract; it represents the income potential associated with servicing the loans.

As with IO strips, the portion of the service fee associated with a particular loan stops when that loan is prepaid. Thus as interest rates fall below the pool contract rate, prepayments speed up, fee income is reduced, and the value of the servicing rights falls. An implication is that a servicing flow contract's duration also can be either positive or negative, depending on the level of current market rates relative to the pool contract rate.

Reverse Annuity Mortgages

Another rate-sensitive fixed-income security with a weird duration, although of a different type, is the reverse annuity mortgage (RAM). Unlike the borrower in a typical loan, who gets a large principal sum from the lender and then makes monthly payments over many years, a RAM borrower receives a series of cash flows from the lender and then, at the end of the stated term, makes a large payment to the lender to retire the loan. Hence the name *reverse annuity*, reflecting the reversal of the traditional payment process.

RAMs were designed to help elderly home owners tap into their accumulated equity without selling their homes. The theory is that an older home owner with significant equity can use a RAM to begin withdrawing that equity systematically by borrowing, rather than all at once by selling the property. With each payment from the lender, the loan balance grows by the amount received, plus interest. For example, suppose a home owner obtains a RAM with a 10% contract rate that pays him $250 per month for 10 years. Each month the lender sends the borrower a check for $250, so at the end of the 10th year the borrower will have accumulated a debt of $51,211, calculated as follows (students of time value will recognize this computation as a *future value of an annuity* application):

$$\$51{,}211 = \$250 \times \frac{\left[1+\frac{.10}{12}\right]^{120} - 1}{\frac{.10}{12}} .$$

Just as with other mortgage loans, the RAM's payment stream can be sold. The value is calculated as the present value of the payment stream (a negative amount for the lender) plus the present value of the repayment to be received in year 10. If the market interest rate stays at 10%, the value of the RAM will be zero (the present value of what the lender gives up is just equal to the present value of what the lender will receive). If, on the other hand, the market rate falls to 9%, this security's value becomes $1,155:

$$\$1{,}155 = \$250 \times \frac{\left[1+\frac{.09}{12}\right]^{120} - 1}{\frac{.09}{12}\left[1+\frac{.09}{12}\right]^{120}} + \frac{\$51{,}211}{\left[1+\frac{.09}{12}\right]^{120}} .$$

When the market rate falls, the present value of the outflows to the home owner is less than the present value of the balance due the lender at the end of the term.

If we calculate this RAM's duration in the Macaulay sense, the solution turns out to be more than 1,000 years! An alternative interpretation is that Macaulay duration calculations have little meaning when applied to fixed interest rate RAMs. This view suggests that an analysis of interest rate risk for these types of RAMs should involve the same issues that apply to IOs and servicing contracts. After all, the life of a RAM depends on future interest rate movements. When rates move up dramatically, the borrower will do everything possible to remain in the residence, since the low-cost money borrowed can potentially be lent at a high rate – creating a virtual *money machine* for the borrower, and a disaster for the lender. Thus we must evaluate fixed-rate RAMs in some type of risk analysis framework, such as *Monte Carlo* simulation. In any event, the high duration causes managing a RAM's interest rate risk to be a truly difficult task.

Hedging

It might seem natural to ask why, aside from knowing how much the price of a security should change for a given change in interest rates, investors should care about duration. The basic reason is that duration is a useful *hedging* tool. Hedging is the process of protecting an investment portfolio from unexpected changes in value. In a classic example, financial institutions are often encouraged to match the durations of their liabilities with the durations of their assets so that their portfolios are *immunized* against changes in interest rates. When asset and liability durations are equal, any increase in liability values caused by a change in interest rates will be just offset by an equal increase in asset values. Firms whose assets and liabilities have unequal durations are exposed to interest rate risk, with greater risk as the difference between durations increases.[7] Thus an investor who does not match asset and liability durations bears the risk that changes in market interest rates may result in the portfolio's earning a return insufficient to fund the liabilities.

One problem associated with using duration to hedge, or immunize, a portfolio is that duration changes as market interest rates change. As we show above, for example, a 30 year, 10% coupon bond will experience a change in duration from 11.411 years to 8.695 years if required market yields shift from 8% to 12%. As a result, the investor must *rebalance*, by either buying or selling securities, every time interest rates change if the portfolio is to remain optimally hedged.[8]

The existence of securities with negative durations points to a different hedging strategy. Recall, from the discussion above, that negative duration implies that a security's price will change in the same direction in which market interest rates (required yields) change. An IO strip thereby provides a useful hedging opportunity, since its price will rise as interest rates rise (if the market rate is below the coupon rate on the loan portfolio), offsetting the price decline on a typical fixed income security. For example, an investment manager could greatly reduce a portfolio's change in value when interest rates change by adding IO strips to the mix of securities. If market interest rates rise, the rise in the IOs' prices will offset the drop in the values of traditional bonds held. If market rates fall, on the other hand, the IOs' price decline is offset by the rise in value realized on the bonds. Given the price characteristics of IO strips, a financial institution is likely to find IOs to be excellent securities for use in hedging its *interest rate gap* (the average rate of return earned on assets minus the average rate paid on liabilities).

Because of the pricing characteristics of mortgage loan servicing flow contracts, a bank that services a large portfolio of home loans has to pay special attention to hedging this position. One security that is particularly useful for hedging a servicing flow contract is a principal-only (PO) strip. Because a PO strip entitles its holder to receive a stated amount of principal, this instrument's value is maximized if required market interest rates plummet and borrowers refinance (and prepay all principal) right away. Thus, PO strips have price characteristics opposite those of IO strips.[9] By combining PO strips in a portfolio with mortgage loan servicing contracts, a bank would be hedged against the negative price pressures placed on servicing flow contracts when market interest rates fall.

Hedging a RAM represents a significantly different problem. With a security that sensitive to interest rate changes, a portfolio manager would be hard pressed to find a security with which to hedge. Therefore, a RAM should either carry an adjustable coupon rate, or else be issued by a member of the borrower's family (to prevent *agency problems* that could otherwise affect the timing of prepayments).

Summary and Conclusions

This paper outlines the concept of duration, with a focus on some weird cases that arise in the real estate financing arena. Duration is a measure of a fixed income security's price sensitivity to changes in interest rates. However, the standard calculation formula does not work for securities, such as home loans and mortgage-backed securities, whose cash flows are unpredictable because of their sensitivity to market interest rate movements. In such cases, we can compute an approximation of duration if we know the security's price reaction to a particular change in interest rates.

Using these approximations can result in a negative duration measure if the price change is positive when interest rates increase. This weird outcome makes some mortgage-related instruments useful hedging tools, in that negative duration offsets the positive durations of the fixed-income securities that portfolio managers typically hold. The unusual duration situation for a security such as the RAM, with its large number of small cash outflows and single large inflow at maturity, can create significant hedging problems. ■

Notes

1. A semiannual rate of 4% would actually be consistent with an annual rate of 8.16%, such that $(1.04)^2 = 1.0816$ with compounding. However, we often simplify illustrations of semiannual discounting by halving the annual rate.
2. The residual on a collateralized mortgage obligation, or CMO, is the difference between the cash flow paid to the CMO investors and cash payments received on the loans serving as collateral. The two other negative duration instruments listed are described more fully in later sections.
3. The implication of hedging with IO strips is discussed in more detail later in this article.
4. Davidson, Andrew S., "Overview of Alternative Duration Measures for Mortgage-Backed Securities", in *Mortgage-Backed Securities*, Frank J. Fabozzi (ed.), Chicago: Probus Publishing, 1987, p. 67-79.
5. Jacob, David P., Graham Lord, and James A. Tilley, "Price, Duration and Convexity of Mortgage-Backed Securities", in *Mortgage-Backed Securities*, Frank J. Fabozzi (ed.), Chicago: Probus Publishing, 1987, p. 81-101.
6. Actually, part of this 50 basis point fee is paid to GNMA for its guarantee (which, in turn, motivates the investors to accept only 9.5% whereas the original lenders had expected more). The remainder of the fee compensates the servicer for collecting and processing monthly payments, delivering the payments to the MBS investors, and taking remedial action when borrowers are delinquent.
7. It should be noted that immunization is an efficient strategy, but may not be the optimal strategy. Immunizing is a good plan for investors with fixed-sum liabilities that have certain maturities; or investors who wish to earn fixed, certain rates over set planning horizons. In the latter case, the investor forces any decrease (increase) in asset values resulting from an increase (decrease) in yields to be exactly offset by an increase (decrease) in reinvestment returns *if* the bond's duration is equal to the chosen holding period.
8. It is also possible to have a "non-optimal" hedge, which reduces, but does not eliminate, value movements. The benefit of reducing volatility must be weighed against higher transaction costs associated with rebalancing a hedge.
9. Of course, a PO is not suited to hedging risk on an IO, as an IO and PO together constitute an entire loan contract.

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Rolling the Dice: Would Casinos Harm Illinois Home Values?

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Casino gambling came to Illinois in February, 1990 when riverboat operations were approved by the General Assembly. At first, ten licenses were granted, each valid for one or two boats up to a total passenger capacity of 1,200. Among the locations for which these initial licenses were granted were sites on the Mississippi River in East St. Louis, on the Illinois River in Peoria, and on the Des Plaines River in Joliet. Authorized games included blackjack, poker, craps, roulette, video poker, keno, baccarat, and slot machines, in addition to various punch board-type games. Riverboat gaming was prohibited in Cook County and anywhere on Lake Michigan.

Boating Enthusiasts

The Illinois Gaming Board's authorization to strictly regulate the industry included such minutiae as the design of the vessels, minimum passenger capacity (500), and the route and duration of each cruise. Yet even though gaming was heavily regulated, the nose of the camel had slipped into the tent, so to speak. Once river-based gambling had been approved, it was only a matter of time before expansion was suggested.

Surely enough, subsequent to this original legislation additional bills were introduced to expand gaming, in terms of both the number of locations and the establishment of land-based operations. Legislation in 1995 sought to expand the number of sites and to add Cook County to the list of permissible locations. It proposed to establish a Metropolitan Entertainment Complex, with five riverboats and land-based entertainment facilities in Chicago. Another bill sought to allow dockside gaming. When the competitive situation worsened for Illinois riverboats operating near gambling-friendly Iowa, bills were introduced to reduce cruising requirements, and to allow dockside gaming as long as the riverboat was based within seventy-five (later fifteen) miles of a competing establishment.

Although none of these bills were adopted, their introduction signaled the willingness of some legislators to loosen the regulation of riverboat gaming and allow expansion. Some of the proposed loosening has come in response to more liberal gambling regulation in nearby states. Cruise and wagering restrictions were lifted in Iowa in mid-1994, and Missouri vessels added slot machines in December of that year. One result has been a decline in the profitability of two gaming vessels based in northern Illinois (*Silver Eagle* and *Casino Rock Island*). Thus, there exists the real possibility that interstate rivalry and competition will lead to ever-increasing pressure on state legislatures to liberalize the regulation of gaming establishments.

Even without the expansion of sites on which gambling is permitted, Illinois gaming receipts have grown exponentially since 1991. The state's gross 1991 gaming revenues were $14.9 million. In 1992, that figure rose to $226.3 million, and a year later gross revenues stood at $605.5 million. By 1994 the figure was $980.7 million, and in 1995 it finally passed the billion-dollar mark, standing at $1,197.3 million. Since the state government has a stake in the gross revenue figures (it takes in a fee of $2 per passenger and 20% of adjusted gross receipts), it has an incentive to allow the expansion of the gaming industry. In contemporary business parlance, the State of Illinois has become a *stakeholder* in legal gambling.

In summary, interstate competition and the desire for added tax revenues may have combined to lead the General Assembly to expand gaming in Illinois. To the extent that expansion proliferates, we might ultimately expect to see the approval of land-based casinos. The possible existence of land-based gaming establishments in Illinois naturally leads to questions regarding the cost/benefit effects of land-based casinos. To address these questions, we must develop a clearer picture of the costs that gambling imposes on the state and its communities, and then make suitable comparisons with the accompanying benefits. The benefits, as typically enumerated by gambling proponents, include tax revenue, higher employment, and general economic growth. To calculate the overall net effect of gaming, the analyst must then subtract the negative impact of the costs typically cited when a jurisdiction considers legalizing gambling: added crime, betting addictions, and families' financial ruin.

Leaving Rock Island? Viva Decatur?

Another critical cost – typically ignored – is the impact that nearby gaming businesses would have on residential property values if land-based casinos were permitted to locate in suburban areas. Earlier *Illinois Real Estate Letter* discussions of legalized gambling ("Illinois Waterfront Development: A Public Gamble," Winter 1997 and "On Golden Ponds and Riverboats," Winter/Spring 1994) addressed interesting issues, but not questions regarding how home values would react to casinos. Indeed, it is difficult to speculate in advance what the potential effect of land-based casinos would be on the values of residential properties in Illinois.

Fortunately, a laboratory exists where this effect can be evaluated. Las Vegas is, arguably, the gaming capital of the world, a city visited by more than thirty million travelers each year. Tourists are familiar with the huge casinos in the "downtown" area and along the "strip," where eight of the nine largest hotels in the world are located. What many people do not realize, however, is that numerous small and not-so-small local casinos are located in the surrounding suburban areas.

For example, in Henderson, Nevada, located between ten and fifteen miles from the Las Vegas "strip," there are a multitude of gaming establishments of various sizes located close to residential areas. These establishments vary in size from taverns with a few slot machines to large casinos with live table games, and they are patronized almost exclusively by local citizens. In fact, these suburban casinos seek to cater to the needs of area residents rather than tourists; they offer check cashing and other local promotions. Since many of these decentralized gaming facilities are located in close proximity to residential developments, we can use transaction information from such areas to estimate what, if any, effect the casinos have had on nearby housing values. Furthermore, since the economy of greater Las Vegas depends heavily on the gaming industry, we would expect that a finding of a *negative* impact in that region could certainly be extrapolated to other areas where gambling would enjoy lower degrees of community acceptance.

Let's Look at the Record

Studies of the effect of *amenities* (good schools, upscale shopping, *etc.*) and dis-amenities, *or nuisances* (waste dumps, power lines, trailer parks, *etc.*) on nearby property values have populated the real estate journals for years. The method of testing for this effect is a well known and accepted statistical technique known as *regression analysis*. An analyst looks at a sample of transactions involving a community where an identified amenity or nuisance exists and, after accounting for differences in the physical and neighborhood characteristics, relates value (as indicated by the observed selling price) to each property's distance from the amenity or nuisance. In other words, it is possible to isolate the effect of distance to an amenity's or nuisance's location from all other factors believed to affect a property's value. If the numerical relationship between price and distance is found to be positive, such that property values increase with greater distance from the specified location, then that location (in this case, the gaming facility) is considered to be a nuisance. Moreover, the size of the numerical relationship (in regression terminology, the *coefficient*) determines the magnitude of the effect; the result need not be "all-or-nothing."

We must also remain mindful of the possibility that neighborhood characteristics other than the amenity or nuisance being studied could impact the values of the properties examined. For example, a shopping center could be located next to a casino, and the shopping center's impact on nearby home values might be as great as, or greater than, that exerted by the casino. Alternatively, a casino might locate on a parcel of land that, for some reason, already has a negative influence on the values of nearby houses. Of course, in such a case it could be difficult to disentangle the effects of these neighborhood influences from that exerted by the casino itself.

Fortunately, we have a method for solving this problem. With access to reliable historical information on prices, we can account for the extraneous neighborhood characteristics by comparing the price-distance relationship that prevailed before the casino came into being with the corresponding figure for the period when the casino was in operation. That is, the distance measure of interest to us in the "before" analysis is distance from the house to the location where the casino was eventually placed. Therefore, only residential properties that had been sold prior to the casino's establishment are included in this sub-sample. Similarly, only houses that were sold after the facility's establishment are included in the "after" sub-sample. We can infer the casino's impact by comparing the "before" and "after" regression results. For example, if the measured relationship between price and proximity is zero in the "before" analysis but negative "after," then it would seem reasonable to conclude that the casino in question is, indeed, a nuisance.

The Supporting Data

To obtain specific measurements of this price-distance relationship, we looked at a sample of 10,762 residential properties that were sold between January 1980 and April 1995. All are located in two zip code areas (89014, 89015) within the Clark County, Nevada community of Henderson. The characteristics of each property that we accounted for are: the number of bathrooms, bedrooms, and fireplaces; the existence of a garage or a pool; the age of the property; the date of sale; the square footage of the building and the lot; and, of course, the distance to the nearest casino site. In addition to dividing the information into a subset of houses sold prior to the establishment of the casinos and a subset containing those sold with existing casinos nearby, we divided the sample into properties that were located close to *smaller* casinos and those located close to *larger* casinos. We have defined an establishment as a "large" casino if it has 100 slot machines or more, and as a "small" casino otherwise. While this division is arbitrary, it allows us to determine whether the impact of a casino varies by size, and what the nature of that impact may be. The sample is large because all information in the Clark County Assessor's office is available on a CD-ROM disk, a medium that allows for quick search and download capabilities. The existence of such a large amount of supporting information gives us more confidence in the statistical results that we obtain.

While it is difficult to speculate on the potential effect of land-based casinos on Illinois home values, a laboratory exists where this effect can be evaluated: Las Vegas, the gaming capital of the world.

The regression technique utilizes a series of *independent* variables in explaining a the magnitude of a *dependent* variable. In this study, the dependent variable is a measure of the real price (in constant dollars) of each residential property. That is, we divided the actual sales price of each included house by the Consumer Price Index (CPI) for the month when the property was sold; if observed *nominal* home prices rose each month in exact step with general inflation, as measured by the CPI, then homes' *real* prices would be unchanged. We then took the *logarithm* of the real price; converting our measured housing prices to "logs," which are exponents to which "base" numbers are raised in reaching specified totals, allows us to interpret our results in terms of percentages rather than dollars. This approach helps us apply our results to communities other than Las Vegas. For example, a reported average price difference of $2,000 per house would not represent a large impact in a Chicago neighborhood with an average home price of $300,000; it *is* a large impact, however, in a neighborhood where the average price is $75,000.

In short, the coefficient that the regression output assigns to each independent variable (age, size, room count, distance to the nearest casino) is interpreted as the percentage change in the price of the home associated with each considered characteristic. In line with accepted statistical techniques, we deem the coefficients to be *significant* (more technically, significantly different from zero) if there is less than a 5% probability that the indicated relationship could have occurred simply by coincidence. That is, the smaller the probability of a chance relationship, the greater confidence we can have that the measured effect is due to the characteristic in question. Because our transaction sample is so large, there is little concern that we could mistakenly interpret coincidental outcomes as meaningful indicators of residential property values. (For example, old houses occasionally sell for more than otherwise-similar newer houses, but that unusual result will not be systematically observed across 10,000 transactions.)

The coefficients that the regression software has computed for all of our variables make sense, with the exception of those on the number of bedrooms and number of bathrooms, for which the coefficient values are negative! At first blush, this result would seem to indicate that a house with more bedrooms or bathrooms should sell for a lower price. However, since we also included the size of each house (in square feet) as an independent variable, negative coefficients on the bedroom and bathroom variables may reflect a discount imposed on properties with bedrooms and bathrooms that are large in number but small in size. While this commonly-encountered statistical problem (known as *multicollinearity*) can cause some interpretation difficulties, it has no impact on the validity of the coefficient computed for the distance-to-casino variables.

Results of the Study

A measure of particular importance in any regression analysis is the percentage of variation in the dependent variable associated with changes in the specified independent variables. A high value for this measure, called the *coefficient of determination* or, more commonly, R^2, would indicate that we have done a good job of identifying physical and locational features that determine home values. Our measured R^2s were fairly high: .78 (for the "before" sub-sample), .82 (the "after" sub-sample involving "small" casinos), and .83 (the "after" sub-sample involving "large" casinos). The latter figure indicates that 83% of the variation in prices of houses located near large casinos relates to the independent variables we chose. The other 17% would be explained by landscaping, fence quality, location on a *cul-de-sac*, or other features we did not include as independent variables (because experience and common sense told us that their impacts would be small, and because added statistical problems can arise as the number of independent variables grows).

We can conclude that casinos are nuisances – they exert negative influences on the values of nearby residences – and that larger casinos create bigger problems.

The coefficient value of greatest interest to us, of course, is that for the distance variable. To test for the differential impact of large *vs.* small casinos, we actually included *two* distance variables. The model is set up so that the effect of distance to any casino is indicated by the coefficient on a variable called MILES, while we capture the added effect of distance to a large casino (we had theorized that larger casinos would exert greater negative impacts on nearby houses) by summing the coefficients on MILES and a variable called CASMILES.

The results indicate that the value of a home sold before a casino is established is not affected by distance to the casino's future location. In other words, no disamenity is systematically associated with sites that ultimately will hold casinos, or with the land uses observed near those sites. After a small casino opened, however, the value of each home within one mile of that casino fell by approximately 3.27%. (If the casino was larger, the value of each house located within one mile fell by approximately 4.6%.) Because our output shows a very small probability that any coefficient could result from chance alone, we have much confidence in these distance measures. Thus, we can conclude that casinos are nuisances – they exert negative influences on the values of nearby residences – and that larger casinos create bigger problems.

We can determine the aggregate impact of having a casino in a residential area by applying the above percentage reductions to all houses located within a mile radius of the casino. For example, if the average value of the 400 residential properties located within a mile of a proposed casino is $200,000, then the total value of all homes would be $80 million. A small casino would have an aggregate negative value impact of $80,000,000 x .0327 = $2,616,000; a large casino would have a negative impact of $3,680,000. An analyst should include this impact in the equation that measures the typically-cited costs and benefits that communities realize through casino gaming.

Conclusions

Obviously the exact impact of any casino in a residential area will vary depending on the city in question. However, results from near Las Vegas suggest that a casino is a nuisance that negatively impacts nearby residential properties, at least those located within a mile of the facility. Illinois citizens are no more likely than their Nevada counterparts to want to live near the lights, noise, and traffic that accompany casino operations. The next time gambling legislation is discussed, they would be justified in asking gaming proponents why the impact on home values in Illinois would be any different from that found in the Henderson study.■

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Vacancy Management IV: Quality of Real Estate Services

Peter F. Colwell

After a truly long hiatus, we return to our series on this crucial property management topic. Readers are referred to prior vacancy management articles in our Summer 1989, Winter 1990, and Spring 1990 issues. A more technical coverage follows this two-page discussion.

The old expression "you have to spend money to make money" is applicable in any business enterprise. It is clear, for example, that a firm must spend more promoting its products or services if its revenues are to increase. Perhaps less obvious is the relationship between cash inflows and outflows, a relationship that is especially important in managing real estate because of that asset class's unique spatial/locational attributes.

Indeed, some real estate managers view *operating statements* only as accounting devices, ignoring the insights these documents can offer into critical management issues. Because the operating statement's simple algebra tells us that potential gross income minus allowances for vacancy and bad debts equals effective gross income (*EGI*), and that *EGI* minus operating expense (*OE*) equals net operating income (*NOI*), a manager might conclude that maximizing *NOI* involves simply keeping *EGI* as high, and *OE* as low, as possible. But *NOI* maximization requires a fuller understanding of fundamental interrelationships associated with the items in the operating statement.

For example, in altering rents we will change not only occupancy levels, but also the tenants' propensity to generate bad debts; operating expenses will change as well. These effects might seem somewhat obvious to anyone who has observed the market for rental real estate. Yet there are other, more complex interrelationships among operating statement items. Notably, changing *OE* in some dimensions affects the *quality* of the services associated with occupancy. Thus rents, occupancy levels, and bad debts may all be affected. Then, the feedback generated by these changes can cause further changes in *OE*. Yet as boggling as the various relationships may seem, they can be illustrated through an analytical device – a model – based on some straightforward economic principles. An important result revealed by our model is that the optimal response to a decrease in demand is to *reduce* quality.

Figure 1

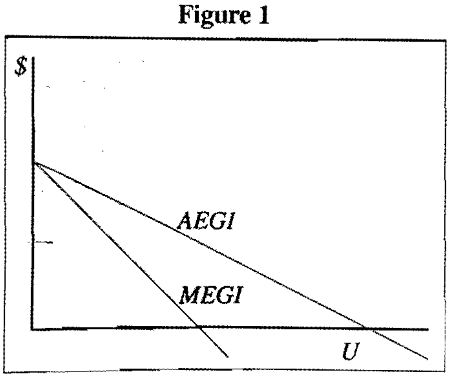
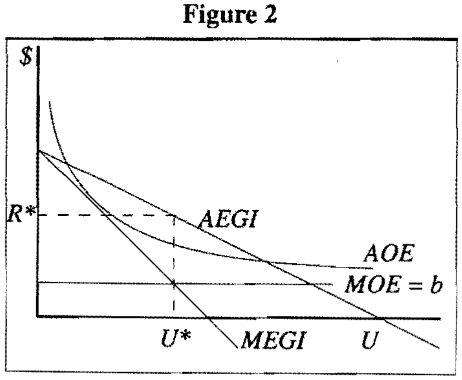


Figure 2



A Simple Microeconomic Model

The behavior of a company that owns an office property can be modeled like that of any other type of firm; the only special attribute is that there is some degree of spatial market power. In other words, the landlord's actions can affect the rent that tenants are willing to pay; an office property is not like a farm commodity whose producer must be a *price-taker*. We can assume that, as more of the property's units become occupied, the firm's *EGI* first increases, but then decreases. *EGI* might be expected to decline over some range of higher occupancy because the manager achieves higher occupancy by reducing rents; if the drop in *EGI* from lower rent more than offsets the increase from higher occupancy, then *EGI* falls.

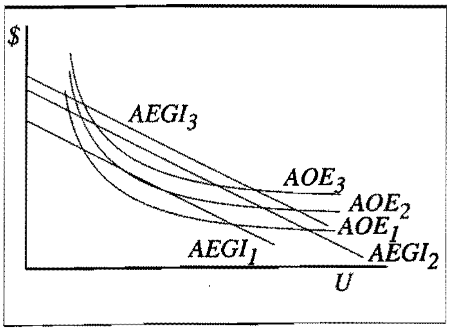
Of course rent per unit, or average *EGI* (*AEGI*), declines with the number of occupied units. Moreover, we might assume that this relationship is *linear*, such that the amount by which rent must fall in order to increase occupancy by one unit remains constant as occupancy increases. We therefore represent the rent relationship (*AEGI*) in Figure 1 as a standard, downward-sloping linear demand curve.

We can also find the *change* in *EGI* that results from a one-unit change in occupancy. Those who have studied basic microeconomics will recognize this relationship as the standard *marginal revenue* curve; we will call it marginal *EGI*, or *MEGI*: the extra revenue that results from adding an additional unit of occupancy. Note that in Figure 1 the slope of the *MEGI* curve is twice as steep as the *AEGI*, or rent, curve (*MEGI* intersects the horizontal axis half as far out as *AEGI*).

The cost side is equally simple. We assume that *OE* is linearly related to the number of occupied units. While we also assume that *OE rises* with occupancy, this relationship does not necessarily have to hold. For example, a landlord might face *declining* expense with higher occupancy if each lease had a common area maintenance (CAM) term, under which the tenants pay to maintain common areas (such that the landlord must pay unoccupied units' share). Ordinarily, however, higher intensity of use brings about higher costs, such that the change in *OE* resulting from a unit change in occupancy (marginal operating expenses, or *MOE*)[1] is positive. Moreover, because we have assumed a linear relationship between occupancy and expenses, *MOE* has a constant value. Figure 2 shows *MOE*, with its constant slope (shown as *b*), along with average operating expenses (denoted *AOE*).

One of the most important lessons from introductory economics is that we maximize profit by equating *marginal revenue* with *marginal cost*. For the firm providing office real estate services, we would maximize *NOI* by equating *MEGI* with *MOE*. If occupancy is less than U^*, the added *EGI* from increased occupancy exceeds the added operating expense (the height of *MEGI* exceeds *b* to the left of U^* in Figure 2), and therefore it is profitable to increase occupancy. However, for

Figure 3



occupancy greater than U^* the addition to *EGI* is less than the accompanying added operating expense (*MEGI* 's height is lower than *b* to the right of U^*); the manager would actually earn a higher profit by *reducing* occupancy.

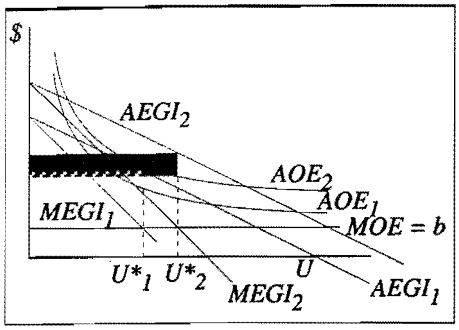
Thus, under conditions represented in Figure 2, optimal occupancy is at U^*, where added operating expenses are just equal to additional *EGI*. The optimal level of rent, R^*, is the *EGI* per unit at optimal occupancy U^*. Optimal rent therefore depends on influences that include the height of the demand curve and the slope *b* of the operating expense function.

A Model That Incorporates Quality

Basic microeconomics focuses on the price/*quantity* relationship, but we can expand our analysis by incorporating a *quality* measure. Suppose that quality is represented by the *height* of the operating expense function; greater height indicates higher service quality regardless of occupancy. For example, replacing light bulbs in hallways on a regular, early schedule creates more cost (shown graphically as a higher *AOE* function) than changing them only after they burn out and tenants complain. Tenants' response to higher service quality should be to shift *AEGI* upward.

We can link the operating expense function's height with that of the demand (rent, or *AEGI*) curve through a model in which we assume that *AEGI*'s *intercept* (rent that results in no occupied units, and zero revenue) increases at a decreasing rate as operating costs rise to reflect higher service quality. In Figure 3, an increase in average operating costs from AOE_1 to AOE_2 causes demand to rise from $AEGI_1$ to $AEGI_2$. But a further increase in average operating costs by the same amount (AOE_2 to AOE_3) causes only a relatively small rent increase, from $AEGI_2$ to $AEGI_3$.

Figure 4



Recall that our goal is to maximize *NOI*, the difference between rent per unit and average operating expenses per unit, multiplied by the number of units rented. In Figure 4, the firm facing rent of $AEGI_1$ (and marginal rent of $MEGI_1$) operates at an occupancy level of U^*_1. With average operating expenses of AOE_1, this firm's *NOI* is shown by the striped rectangle. What happens if the firm improves service quality such that average operating cost rises to AOE_2? Rent rises to $AEGI_2$, and marginal revenue rises to $MEGI_2$. (Since we assume that the intercept of the operating expense function increases, but not the slope, *MOE* is unaffected by the operating expense increase). The firm's optimal occupancy level increases to U^*_2, and *NOI* is equal to the area represented by the shaded rectangle.

In this example, it seems clear that it is in the best interest of the firm to operate at higher-cost occupancy level U^*_2 rather than at U^*_1, since this higher occupancy yields a higher *NOI*. But will additional, similar increases in service quality (and operating costs) continue to generate higher *NOI*s? The answer is no, because of the manner in which rent responds to changes in operating costs: at some point *NOI* per unit will decrease with continued increases in quality (our treatment of *AOE* as rising more rapidly than *AEGI* should be intuitively appealing). Eventually, the *loss* from a lower *NOI* per unit on all occupied units will exceed the *gain* from increasing the number of units occupied.

One notable result that can be offered from a more detailed version of this analysis is that a decrease in demand (represented by a lower vertical intercept at each level of quality, or by a steeper slope) will reduce the optimal level of quality (see Table 1). The appropriate response to a decrease in demand (from an *NOI* standpoint) is to decrease quality. Yet conventional wisdom tells some managers to increase quality when demand decreases, to reduce the vacancy impact on the property. This potential for erroneous conclusions shows precisely why it is helpful to use models in analyzing these complex relationships. Of course, the model presented in this discussion may be flawed. If so, others may wish to suggest alternative models that deliver results more in line with the conventional wisdom on the market's approprate quality response to a decrease in demand.

In conclusion, while we do not expect property managers to draw the kinds of graphs shown above, we hope that in understanding the microeconomic principles underlying the relationship between operating expenses and rent they will enhance their firms' performances. ■

1. As a 10th anniversary gift to our more careful readers, we offer the observation that the *MOE* ratio is related to the Current Unrealized Revenue Loss per Year (*CURLY*), Level Average Revenue Realized per Year (*LARRY*), and Shareholders' Effective Monthly Profit (*ShEMP*) measures.

Table 1

$$\frac{EGI}{U} = \alpha - \beta U \qquad OE = a + bU \qquad \alpha = \frac{\ln a - \ln a_0}{r}$$

		base case	DECREASE DEMAND: steeper slope	increase a_0	increase r
	a_0	1,000	1,000	1,100	1,000
	b	2,000	2,000	2,000	2,000
	β	500	550	500	500
	r	.0003	.0003	.0003	.00031
	U^*	9.5	8.2	9.0	8.8
	NOI^*	13,625	9,706	10,677	10,274
QUALITY INDEX	a^*	31,755	27,367	30,104	28,353

A Technical Analysis of the Quality of Real Estate Services

We have just seen that a firm acting optimally will select that level of rent, and therefore of occupancy, for which marginal operating expense is just equal to marginal effective gross income. The firm's resulting net operating income (*NOI*) was shown to be rent minus average operating expense at the optimal occupancy level, multiplied by the number of occupied units. This outcome was the best that the firm, facing particular demand and marginal operating expense curves, could attain. Of course, when we considered the relationship between the quality of real estate services and the level of rent, we saw that the firm potentially realizes many different optimal *NOIs*, one for each level of service quality. The key question therefore becomes which of the potentially optimal *NOIs* is *truly* optimal.

To identify this true optimal *NOI*, we must first specify the way in which rent responds to changes in the quality of real estate services. Second, we must identify precisely how average operating costs respond to changes in the quality of real estate services. Finally, after incorporating some basic mathematical equations, we can identify the truly optimal *NOI*. We can begin by reviewing the analytical framework of the previous article, while providing an extra measure of technical detail along the way.

Seeking the Optimum

In the upper right quadrant of Figure 1, rent, or average effective gross income (*AEGI*), is represented as a downward-sloping straight line. The relationship is shown as linear because the relationship between *EGI* and the proportion of units that are occupied is assumed to first increase and then decrease, in a manner represented graphically as a *parabola*, as the occupancy level increases. This result can be expressed mathematically as:

$$EGI = \alpha U - \beta U^2 ,$$

an equation in which both α and β are greater than zero and U is the number of occupied units. Dividing both sides by the number of occupied units, U, results in the rent, or *AEGI*, function's being a straight line:

$$R = AEGI = \alpha - \beta U ,$$

an equation in which α is the vertical intercept and β is the slope, or steepness, of this demand "curve." Marginal effective gross income (*MEGI*), which has the same intercept as *AEGI* but a slope that is twice as steep, is the change in *EGI* attributable to a one-unit change in the number of occupied units. *MEGI* is computed as the *first derivative* of the *EGI* function:

$$\frac{dEGI}{dU} = \alpha - 2\beta U .$$

Operating expenses are assumed to increase along a straight line as the number of occupied units increases:

$$OE = a + bU ,$$

an equation in which both a and b are, again, assumed to be positive. Marginal operating expense, or the change in total operating expenses attributable to a one-unit change in occupancy, is computed as the following constant:

$$\frac{dOE}{dU} = \mathrm{b} .$$

The optimal occupancy is the level for which marginal operating expenses are just equal to marginal effective gross income. This relationship implies that

$$\alpha - 2\beta U^* = b ,$$

an equation for which U^* is the level of occupancy that maximizes *NOI*. Solving for U^* allows us to find the number of occupied units that maximizes *NOI*:

$$U^* = \frac{\alpha - b}{2\beta}$$

Substituting the optimal occupancy into the rent (demand) function, we find the optimal rent:

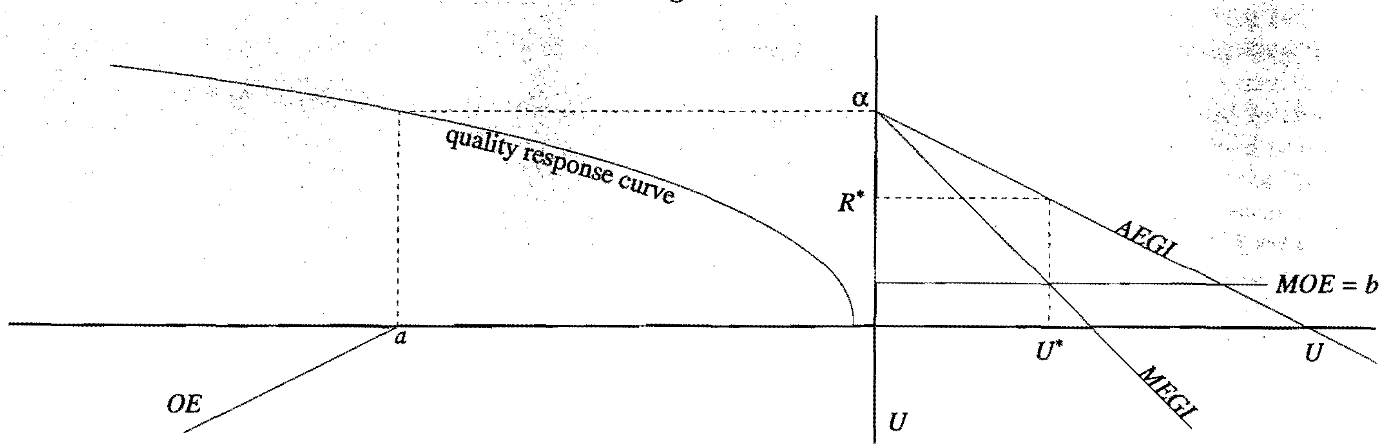
$$R^* = \alpha - \frac{\alpha - b}{2} .$$

Note that rent is one of the primary decision variables in this model. Now that we have reestablished the way in which optimal occupancy and rent are determined, let us turn to the focus of this technical discussion: the effect of changes in the quality of real estate services on *NOI*.

Total Operating Expenses & Demand

Total operating expenses, *OE*, are shown in the lower left quadrant of Figure 1.

Figure 1



A more conventional way to represent operating expenses graphically would be as a positive, increasing function in a quadrant for which the number of units appears on the horizontal axis and price is shown on the vertical axis. In the lower left quadrant of Figure 1, this conventional picture has been inverted, such that units appear on the vertical axis and price is on the horizontal axis. The intercept of the operating expense function, denoted a, can be thought of as the level of operating expenses incurred with 100% vacancy (0% occupancy).

Now, recall the relationship we have assumed between rent and operating expenses: that the intercept of the rent, or *AEGI*, function increases at a decreasing rate as the intercept of the operating expense function, a, increases. The *quality response curve* in the upper left quadrant of Figure 1 shows the intercept of the demand curve, α, that corresponds to the intercept of the operating expense function, a. The fact that the quality response curve increases at a decreasing rate with increases in a can be represented symbolically as:

$\alpha = f(a)$

where $f'(a) > 0$ and $f''(a) < 0$.

The meaning of the first statement is that the height of the rent function, α, depends on the height of the operating expense function (in Figure 1, the horizontal intercept a). The first portion of the latter statement, $f'(a) > 0$, simply means that as a increases, α increases. The second portion, $f''(a) < 0$, means that the size of the increase in α gets smaller with each successive increase in a. (Using mathematical terminology, we would say that the first derivative is positive and the second derivative is negative.) In summary, the height of the demand curve, *AEGI*, increases at a decreasing rate with increases in the height of the operating expenses function.

Average and Total Operating Expenses

Recall the definition of net operating income: rent per unit minus average operating expense per unit, multiplied by the number of units occupied. As discussed in the previous article, average operating expenses increase as service quality improves; this situation is shown graphically as an increase in the height of the operating expense curve, a. Yet while we can observe average operating expenses, how do we know exactly what the level of quality, α, is?

Figure 2 demonstrates how, given a particular average operating expense curve, we can detect the level of quality, and consequently the height of the *AEGI* (or demand) curve associated with this level of quality. In that figure, the firm faces average operating expense curve $AOE(a^{\dagger})$. The meaning of this notation is that average operating expenses depend on the horizontal intercept of the operating expense curve ($a^{\dagger}$, a magnitude that is yet to be identified). Let us randomly select some level of occupancy in Figure 2, perhaps four units. At this occupancy level, we know what average operating expenses are, and we also know what marginal operating expenses are. Now, recall the formula for computing operating expenses:

$$OE = a + bU.$$

Dividing each side of this equation by the number of occupied units, we can compute average operating expenses, or operating expenses per unit, as

$$\frac{OE}{U} = \frac{a}{U} + b.$$

Rearranging algebraically, we can see that a divided by the number of units must be equal to

$$\frac{a}{U} = \frac{OE}{U} - b.$$

If $U = 4$, then

$$\frac{a^{\dagger}}{4} = AOE^{\dagger} - b.$$

In Figure 2, we can identify $a^{\dagger}$ as four times the difference between average and marginal operating expenses at a four-unit level of occupancy.

Another trick to identify $a^{\dagger}$ is to draw a line that emerges from the vertical axis at b, with a slope equal to one divided by the number of units at which *AEGI* is measured. The result is shown in Figure 2 as the dark line with a slope equal to one-fourth. Given average operating expenses that prevail at four units of occupancy, we can read horizontally from the height of $AOE(a^{\dagger})$ to the point where this new line is crossed; we can verify visually that the resulting horizontal distance is $a^{\dagger}$. We can reach this same result algebraically

Figure 2

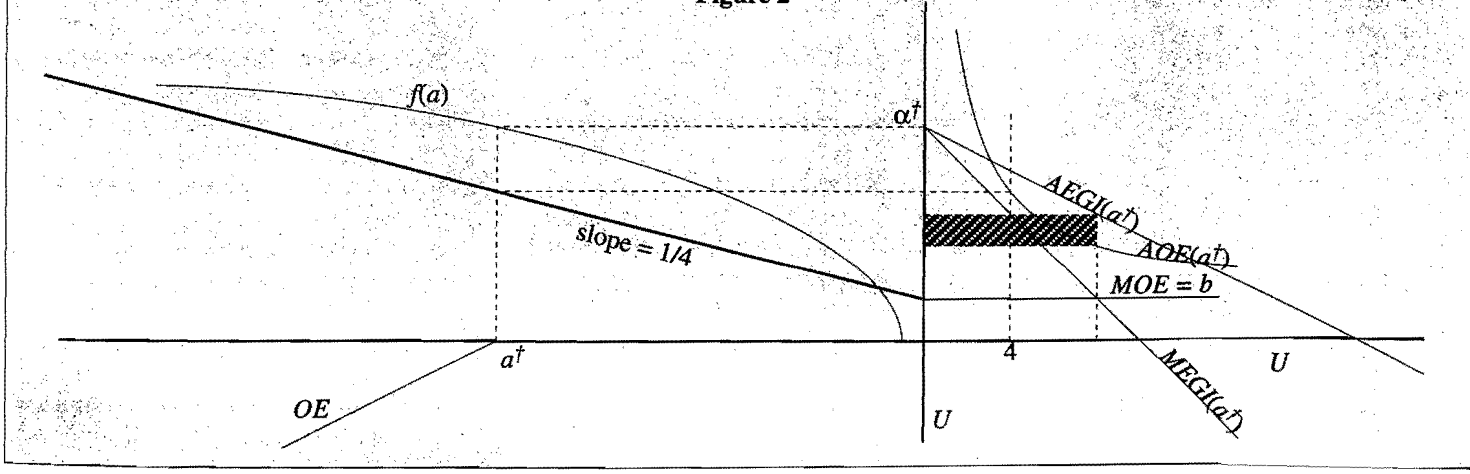
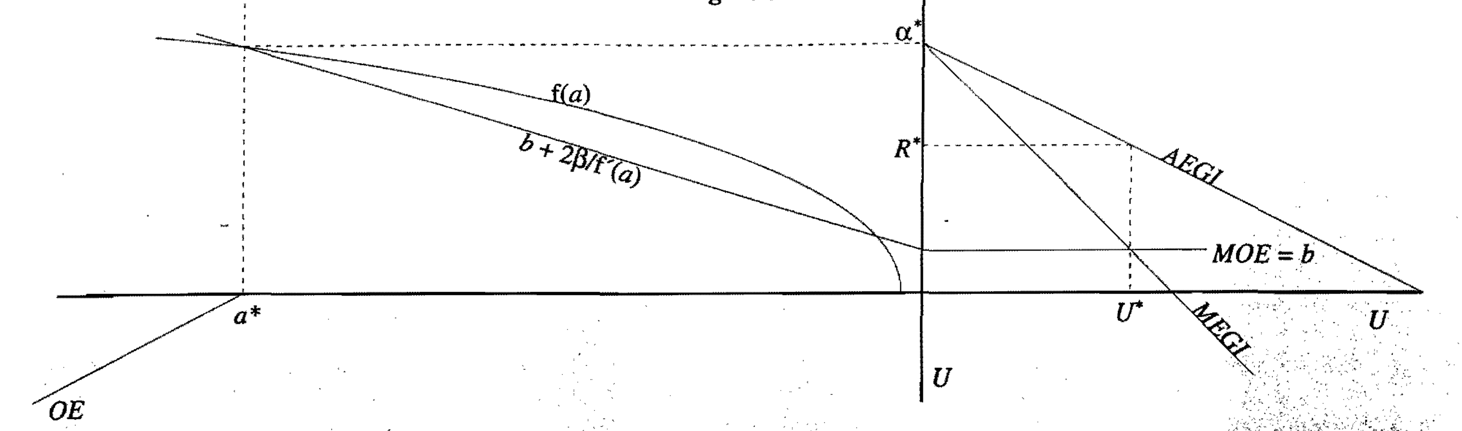


Figure 3



by rearranging terms in the previous equation to show

$$AOE^{t} = \frac{1}{4}a + b\,.$$

Therefore, when an increase in total operating expenses causes average operating expenses to increase, we can observe this average amount at four units in order to determine the level of quality, a. Of course, we could have picked any number of units – for example, eight – at which to observe average operating expense. The slope of our line emerging from b would then be one-eighth, rather than one-fourth.

Now that we have identified a^{t} in Figure 2, we can use the quality shift function, $f(a)$, to identify the intercept of the rent function. We can then select the optimal level of rent, and the optimal level of occupancy, for a specified average and marginal effective gross income, and then calculate the maximum *NOI* associated with this level of service quality. This *NOI* level is represented graphically as the shaded rectangle in Figure 2.

If average operating expenses were higher at every level of occupancy, we would observe a new *AOE* curve corresponding to a higher level of quality of real estate services. But an increase in the level of quality, and therefore in observed operating costs, does not necessarily decrease total profit. Although there is a decline in net operating income *per unit*, the overall loss in *NOI* on previously occupied units (for which higher rents had previously been paid) is less than the increase to *NOI* that the manager realizes from the additional occupancy of units.

The Maximum Optimal *NOI*

The firm attains its optimal *NOI* when it sets rent per unit equal to the value indicated along the demand function, at the level of occupancy for which marginal operating expense (*MOE*) is just equal to marginal effective gross income (*MEGI*). Of course, we have established that changes in the quality of real estate services affect both average operating expenses and the level of demand, such that there is a unique optimal *NOI* for every level of quality. We can identify the maximum from among all of the potential optimum *NOIs* by working with the definition of net operating income:

$$NOI^{*} = EGI - OE$$

or, substituting from our definitions of *EGI* and *OE*:

$$NOI^{*} = \alpha U^{*} - \beta U^{*2} - [a + bU^{*}]\,.$$

Next, substituting the optimal occupancy, U^{*}, and the link between the intercepts of the rent and expense functions, $\alpha = f(a)$, into the *NOI* function and expanding produces the following relationship:

$$NOI^{*} = \frac{f^{2}(a)}{4\beta} - \frac{2bf(a)}{4\beta} - a - \frac{b^{2}}{4\beta}\,.$$

This equation describes the *NOIs* that are optimal in light of each possible value for the intercept, a, of the operating expense function. However, at this stage of our analysis we still do not know how to choose the optimal level, a^{*}, of the operating expense function. In order to determine a^{*}, we must discover how NOI^{*} changes as a changes, and then set that level of change equal to zero. In this way (again, computing a first derivative), we can find the maximum from among all the optimal *NOIs*:

$$\frac{dNOI^{*}}{da} = \frac{f(a)f'(a)}{2\beta} - \frac{bf'(a)}{2\beta} - 1\,.$$

We simplify this relationship algebraically, and set both sides of the equation equal to zero such that:

$$f(a)f'(a) - bf'(a) = 2\beta$$

Rearranging so that we are able to show the optimal intercepts of both the rent function and the operating expense function produces

$$f(a) = b + \frac{2\beta}{f'(a)}$$

Of course, the left-hand side of this equation is simply our quality response curve, $f(a)$. Figure 3 provides a graphical representation of the right-hand side of this equation. Note the following problem: the quality response curve and the curve representing the right-hand side of the above equation intersect at two different points. One of these points generates the *minimum NOI*, while the other generates the *maximum NOI*. The *OE* curve and rent functions shown in Figure 3 are the ones that are associated with the maximum *NOI*. The optimal level of quality is a^{*}, and the optimal height of the rent function is α^{*}. Corresponding to these values, the optimal level of occupancy for the firm is U^{*}, and the optimal rent to charge per unit is R^{*}. These particular values of occupancy and rent will yield the *maximum* optimal net operating income to the firm from among all the potential optimal *NOIs*. ■

(continued from the back page)
focus on opportunity costs. However, the radio host's advice is overly simplistic.

First, the comparison should be only with a savings instrument free of default risk, such as a US Treasury bond or an FDIC-insured bank deposit, and having a maturity similar to the loan's expected holding period. Otherwise, the borrower is comparing across risk classes, because a home loan is a risk-free instrument (the payment is *certain* to be owed the next month). Therefore, a decision on whether to avoid paying $200 extra per month on an 8% loan in order to put that $200 into a risky stock mutual fund that has earned 25% in each of the past few years is not a simple computational question (as might be, for example, a decision on whether to incur new loan origination costs in order to refinance at a lower rate). Another risk issue to note is that prepaying closes out valuable *options* that the borrower holds.

Yet also missing from the radio host's analysis was any mention of the federal income tax implications of prepaying the loan in full. A family can subtract interest paid on a home loan from *adjusted gross income* in computing the *taxable income* on which federal income tax is paid. This deductibility causes the net cost of home loan interest to be less than its gross total. Of course, there is also a tax effect on the income side; anyone who takes money from savings to prepay a mortgage loan (in part or in full) loses deductibility of the loan interest, but also has less savings interest on which to *pay* income tax. In the unlikely case of a borrower who owed $80,000 on an 8% mortgage loan but also had $80,000 in an 8% savings plan (not needed for liquidity or other portfolio concerns), it might seem that paying off the loan would be essentially a wash. Actually, an Illinois taxpayer might *gain* by prepaying the loan under these conditions, because Illinoisans are taxed at the state level on savings interest income, but do not benefit from deductions they itemize on their US income tax returns.

Meeting High Standards

Still, we must not forget the important role that home loan interest plays in bringing the American taxpayer's total deductible expenses above the *standard deduction* ($6,900 for married filers in 1997). Without mortgage loan interest in the mix, it is unlikely that the typical household could have sufficient deductible expenses to benefit from *itemizing*. Thus, there would be no marginal benefit, in the form of income tax saved, for some expenses – notably, gifts to charities – typically thought of as "deductible."

Consider a family of four thinking of prepaying its 8%, $80,000 home loan by taking $80,000 (perhaps inherited) out of a 6% savings account. Its adjusted gross income will be $50,000 in the next year if it prepays, or $54,800 (.06 x $80,000 = $4,800 more) if it leaves the $80,000 in savings. Interest income lost by depleting savings would be $4,800 minus 31% (state + federal) tax that would have been paid, for a $3,312 after-tax cost. Loan interest saved would be approximately .08 x $80,000 = $6,400 (actually $6,376 with the systematic drop in principal), minus 28% federal tax no longer saved, for a $4,591 after-tax gain. The family seems to net $4,591 – $3,312 = $1,279 over the next year by paying off the loan.

But what if its other "deductible" expenses were $1,644 state income tax (3% of $54,800), $2,250 property tax, and $2,450 charitable gifts? With $6,376 in mortgage loan interest, the deductible total would be $12,720. Absent prepayment, federal taxable income would be $54,800 – $12,720 – $10,600 (4 x $2,650 personal exemptions), or $31,480, for US income tax (1997 IRS table) of $4,721 and a federal + state total of $6,365.

With prepayment (and no interest to deduct), the family would claim the standard deduction instead of the itemized $6,200 total (state income tax would be $1,500 without $4,800 extra savings interest income). Taxable income would be $50,000 – $6,900 – $10,600 = $32,500, with a federal income tax of $4,879 and a $14 *higher* federal + state total of $6,379; since prepaying does not affect net worth on its balance sheet, this family is better off by *not* prepaying. (The Illinois *credit* for property tax paid would be equal in the two cases, and thus would not change the ranking.) The outcome obviously depends on the specifics of the case at hand, and results could differ in later years as interest payable on the loan continues to fall. But the prepayment decision is hardly a "no-brainer." ■

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To Pre(pay) or Not To Pre(pay): That Is the Question

Joseph W. Trefzger

Few among us would envy someone who can barely budget for her monthly mortgage loan payment. However, there is an ironic comfort in *not* having to wonder whether the early repayment of part or all of a home loan's principal would be financially advantageous. The American borrower who can afford to *prepay* some or all of her loan's remaining principal balance may feel that she faces a quandary almost as serious as that which befell the Bard's troubled Danish prince.

New Issues, Same Bad Advice

The question of early loan repayment is one that continues to weigh on American home owners' minds, although the popular perception appears to have undergone a minor evolution. A few years ago consumerists' advice seemed to focus on total interest paid; the borrower was told that by reducing her home loan balance she could save tens of thousands of dollars over the scheduled amortization period. But efforts such as those of U of I professor Philip Rushing ("The Economics of Accelerated Principal Repayment," *ORER Letter/Illinois Real Estate Letter*, Winter 1991) reminded people that it is not simply total interest dollars paid, but rather the borrower's *opportunity cost*, that determines the soundness of any prepayment decision. For example, it would make no sense to incur additional credit card debt (at 18% nondeductible interest) to allow more to be paid each month on a home mortgage loan (at 8% deductible interest), no matter how many fewer *dollars* in interest would be paid over the life of the mortgage loan.

What we seem to hear from some commentators today is that the key is the interest rate differential: if available funds can be invested in an alternative medium to earn a return higher than the interest rate on the mortgage loan, we are told, then prepayment does not make sense. A financial commentator advised a recent radio program caller that if she could take money from a 6% savings certificate to repay the remaining balance on an 8% mortgage loan, the decision should be a "no-brainer," in that curtailing an 8% outflow is preferable to continuing a 6% inflow, whereas if the alternative outlet were a mutual fund yielding 12% it would be better to incur the 8% expense while preserving the 12% inflow. This analysis is an improvement over many market observers' earlier focus on total interest paid, in that it is an *attempt* to

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